***PARTS OF A CIRCLE***

***Summary:***

***1.*** *The following are the main parts of a circle with centre* ***O***

***θ***

***P***

***•***

***O***

***A***

***B***

***r***

***r***

***Q***

***(i) AB*** *is called an arc*  ***(ii)******OAB*** *is called a sector*

***(iii)*** *line* ***AB*** *is called a chord* ***(iv)*** *line* ***PQ*** *is called a tangent*

***(v)*** *The shaded part is called a segment* ***(vi)******θ*** *is the angle subtended by an arc*

***(vii)******OA = OB =*** *radius of the circle*

***2. Theorem:*** *Since* ***OA = OB,*** *then* ***OAB*** *is an isosceles triangle with its line of symmetry bisecting chord* ***AB***

***3.*** *The following formulas are used in relation to the above circle****:***

***(i)*** *Circumference*  ***(ii)*** *Area* 

***(iii)*** *Arc length* ***AB*** ***(iv)*** *Sector area* ***OAB*** 

***(v)*** *Segment area* ***= sector area − triangle area***

***EXAMPLES:***

***1.*** *Find the length of an arc which subtends an angle of* ***126°*** *at the centre of a circle of radius* ***14cm.*** *Find also the length of the major arc*

***2.*** *A chord is* ***8cm*** *away from the centre of a circle of radius* ***17cm.*** *Find the****:***

***(i)*** *length of the chord*

***(ii)*** *size of the angle subtended by the chord at the centre*

***3.*** *A chord of length* ***10cm*** *is* ***12cm*** *away from the centre of the circle****.*** *Find the****:***

***(i)*** *radius of the circle*

***(ii)*** *length of the minor arc*

***(iii)*** *area of the minor sector*

***(iv)*** *area of the minor segment*

***4.*** *A sector of a circle of radius* ***12cm*** *has an angle of* ***150°*** *at the centre****.*** *The sector is folded to form a cone****.*** *Find the radius of the circular end of the cone*

***5.*** *Find the distance between two parallel chords of lengths* ***32cm*** *and* ***24cm*** *which lie on opposite sides of the centre in a circle of radius* ***20cm***

***6.*** *in the circle below****,*** *sector* ***OAB*** *has a radius of* ***7cm*** *and subtends an angle of* ***120°*** *at its centre* ***O.***

***7cm***

***120°***

***•***

***O***

***A***

***B***

*Find the****:***

***(i)*** *shortest distance of chord* ***AB*** *from the centre*

***(ii)*** *perimeter of the shaded segment*

***(iii)*** *perimeter of the region enclosed between chord* ***AB*** *and the major arc*

***(iv)*** *area of the shaded segment*

***7.*** *The length of the common chord of two intersecting circles of radius* ***10cm*** *and* ***17cm*** *is* ***6cm.*** *Find the****:***

***(i)*** *angle subtended by the chord at the centre of the two circles*

***(ii)*** *area common to the two circles*

***8.*** *Two equal circles of radius* ***5cm*** *intersect at right angles****.*** *Find the****:***

***(i)*** *distance between the centres of the two circles*

***(ii)*** *area common to the two circles*

***9.*** *A sector of a circle of radius* ***25cm*** *has an angle of* ***100⋅8°*** *at the centre****.*** *The sector is folded to form a cone****.*** *Find the****:***

***(i)*** *radius of the circular end of the cone*

***(ii)*** *height of the cone*

***(iii)*** *volume of the cone*

***(iv)*** *total surface area of the cone*

***HINT: (iv) T⋅S⋅A****since its circular end is open*

***10.*** *The minor segment of a circle has a height of* ***4cm*** *and a chord of length* ***16cm.*** *Find the****:***

***(i)*** *radius of the circle*

***(ii)*** *area of the segment*

***11.*** *A dog tied on a rope* ***5m*** *long is tethered to a tree* ***3m*** *from a straight path****.*** *For what distance along the path is one in danger of being bitten by the dog****?***

***EER:***

***1.*** *A chord of length* ***70cm*** *subtends an angle of* ***120°*** *at the centre of the circle****.*** *Find the****:***

***(i)*** *radius of the circle*

***(ii)*** *distance of the chord from the centre*

***(iii)*** *area of the minor segment*

***2.*** *A chord of length* ***6cm*** *is* ***4cm*** *away from the centre of the circle****.*** *Find the circumference of the circle*

***3.*** *A dog tied on a rope* ***2⋅5m*** *long is tethered to a tree* ***2m*** *from a straight path****.*** *For what distance along the path is one in danger of being bitten by the dog****?***

***4.****A sector of a circle of radius* ***10cm*** *has an angle of* ***100°*** *at the centre****.*** *Find the*

***(i)*** *perimeter of the sector*

***(ii)*** *area of the minor segment*

***5.*** *A chord of length* ***6cm*** *makes an angle of* ***40°*** *with the radius of the circle****.*** *Find the circumference of the circle*

***6.*** *A chord* ***3⋅5cm*** *away from the centre of the circle subtends an angle of* ***120°*** *at its centre****.*** *Find the area of the major segment*

***7.*** *The chord of a circle of radius* ***10cm*** *subtends an angle of* ***120°*** *at its centre****.*** *Find the perimeter of the region enclosed between the chord and the major arc*

***8.*** *Find the distance between two parallel chords of lengths* ***32cm*** *and* ***24cm*** *which lie on the same side of the centre in a circle of radius* ***20cm***

***9.*** *A sector of a circle of radius* ***12⋅5cm*** *has an angle of* ***100⋅8°*** *at the centre****.*** *The sector is folded to form a cone****.*** *Find the****:***

***(i)*** *radius of the circular end of the cone*

***(ii)*** *height of the cone*

***(iii)*** *volume of the cone*

***(iv)*** *total surface area of the cone*

***10.*** *Find the distance between two parallel chords of lengths* ***24cm*** *and* ***10cm*** *which lie on opposite sides of the centre in a circle of radius* ***13cm***

***11.*** *In the figure below****,*** *find the shaded area bounded by two concentric arcs*

***2⋅1cm***

***4⋅2cm***

***30°***

***12.*** *In the triangle* ***OBC*** *below****,*** *arc* ***AC*** *subtends an angle of* ***30°*** *at the centre* ***O*** *of a circle of radius* ***4⋅2cm***

***4⋅2cm***

***2⋅8cm***

***A***

***30°***

***B***

***O***

***C***

*If* ***AB = 2⋅8cm,*** *find the area of the shaded region*

***13.*** *The length of the common chord of two intersecting circles of radius* ***28cm*** *and* ***20cm*** *is* ***30cm.*** *Find the****:***

***(i)*** *angle subtended by the chord at the centre of the two circles*

***(ii)*** *area common to the two circles*

***14.*** *The distance between the centres of two intersecting equal circles of radius* ***5cm*** *is* ***8cm.*** *Find the****:***

***(i)*** *length of the common chord of the two circles*

***(ii)*** *area common to the two circles*

***15.*** *Two circles with centres* ***O*** *and* ***C*** *and radius* ***9cm*** *and* ***6cm*** *intersect at points* ***A*** *and* ***B*** *as shown*

***•***

***•***

***O***

***C***

***A***

***B***

***9cm***

***6cm***

*Given that the distance between* ***O*** *and* ***C*** *is* ***13cm,*** *find the****:***

***(i)*** *reflex angle* ***AOB***

***(ii)*** *length of chord* ***AB***

***(iii)*** *area of the shaded region*

***ANGLES IN A CIRCLE***

|  |  |
| --- | --- |
| ***Angles diagram***  ***x***  ***x***  ***A***  ***B***  ***P***  ***Q*** | ***Circle theorems*** |
|  | *The angles subtended at the circumference by the same arc are equal*  ***∴ ∠APB = ∠AQB***  ***In short:***  *Angles in the same segment are equal* |
| ***2θ***  ***O***  ***P***  ***B***  ***A***  ***θ***  ***B***  ***A***  ***P***  ***2θ***  ***O***  ***θ*** | *The angle subtended at the centre by an arc is twice the angle it subtends at its circumference*  ***∴ ∠AOB = 2∠APB***  ***In short:***  *Angle at the centre is twice the angle at the circumference* |
| ***Diameter*** | *The angles subtended at the circumference by a semi−circle is* ***90°*** |
| ***x***  ***y*** | *The opposite angles of a cyclic quadrilateral add up to* ***180°***  ***∴ x + y = 180°*** |

***EXAMPLES:***

***1.*** *In the figure below****, O*** *is the centre of the circle and* ***∠AOC = 130°.*** *Find* ***∠ABC***

***130°***

***B***

***A***

***C***

***O***

***Soln:***

*∠AOC = 2∠ABC*

*⇒ 130° = 2∠ABC*

***∴*** *∠ABC* ***= 65°***

***2.*** *In the circle below****,*** *find the size of angle marked* ***x***

***D***

***C***

***B***

***x***

***60°***

***75°***

***A***

***Soln:***

*∠BAC = x (angles in the same segment)*

*⇒ x = 180° − (60° + 75°) (angles in a triangle)*

***∴ x = 45°***

***3.*** *In the circle below* ***O*** *is its centre* ***∠ABO = 36°***  *and* ***∠OAC = 32°.*** *Find* ***∠ACB*** *and* ***∠OBC***

***C***

***B***

***A***

***O***

***36°***

***32°***

***Soln:***

***∇AOB*** *is isosceles since OA = OB*

*⇒* ***∠AOB*** *= 180° − (2 × 36°) =* ***108°*** *(angles in a triangle)*

*∠AOB = 2∠ACB*

*⇒ 108° = 2∠ACB*

***∴ ∠ACB*** *=* ***54°***

***Also:***

***C***

***B***

***A***

***O***

***36°***

***32°***

*⇒* ***∠AOC*** *= 180° − (2 × 32°) =* ***116°*** *(angles in a triangle)*

*∠BOC = 360° − (108° + 116°) =* ***136°*** *(angles at a point)*

*⇒ 2∠OBC = 180° − 136° (angles in an triangle)*

***∴ ∠OBC*** *=* ***22°***

***4.*** *In the circle below* ***O*** *is its centre and* ***∠AOC = 150°.*** *Find the size of angle marked* ***x***

***B***

***C***

***A***

***O***

***150°***

***x***

***Soln:***

*Reflex angle ∠AOC = 360° − 150° =* ***210°*** *(angles at a point)*

*⇒ ∠AOC = 2∠ABC*

*210° = 2∠ABC*

***∴ ∠ABC*** *=* ***105°***

***5.*** *In the figure below,* ***O*** *is the centre of the circle of radius* ***7cm*** *and* ***∠BAC = 52°.***

***52°***

***O***

***C***

***B***

***A***

*Find the****:***

***(i)*** *size of angle* ***ACB***

***(ii)*** *lengths of* ***AB*** *and* ***BC***

***Soln:***

***(i) ∠ABC*** *=* ***90°*** *(angle in a semi−circle)*

***∴ ∠ACB*** *= 180° − (90° + 52°) =* ***38°*** *(angles in a triangle)*

***(ii)*** 

***∴*** *AB =* ***8⋅6193cm***

***Also*** 

***∴*** *AB =* ***11⋅0322cm***

***6.*** *In the circle below* ***O*** *is its centre****.*** *Find the size of angle marked* ***x***

***C***

***B***

***A***

***50°***

***O***

***26°***

***x***

***D***

***Soln:***

***(i) ∠ABC*** *=* ***90°*** *(angle in a semi−circle)*

***∴ x*** *= 180° − (90° + 50° + 26°) =* ***14°*** *(angles in a triangle)*

***7.*** *In the circle below* ***O*** *is its centre****.*** *Find the size of the angles marked* ***x*** *and* ***y***

***B***

***C***

***A***

***O***

***y***

***D***

***x***

***2x***

***Soln:***

*x + 2x = 180° (angles in a cyclic quadrilateral)*

***∴ x*** *=* ***60°***

***Also******y*** *= 2x = 2(60°) =* ***120°***

***8.*** *In the circle below****,*** *find the size of the angles marked* ***x*** *and* ***y***

***120°***

***y***

***x***

***98°***

***Soln:***

*x + 98° = 180° (angles in a cyclic quadrilateral)*

***∴ x*** *=* ***82°***

***Also*** *y + 60° = 180° (angles in a cyclic quadrilateral)*

***∴ y*** *=* ***120°***

***9.*** *In the diagram below****, C*** *and* ***O*** *are centres of two intersecting circles****.******∠ CPT*** *=* ***38°***

***O***

***•***

***•***

***C***

***P***

***Q***

***R***

***S***

***T***

***U***

***y***

***38°***

***x***

*Find the size of the angles marked* ***x*** *and* ***y***

***Soln:***

***(i) ∠UTP*** *=* ***90°*** *(angle in a semi−circle)*

***∴ x*** *= 180° − (90° + 38° ) =* ***52°*** *(angles in a triangle)*

***Also:*** ***∠POT*** *= 180° − (2 × 38°) =* ***104°*** *(angles in a triangle)*

*y + 104° = 180° (angles in a cyclic quadrilateral)*

***∴ y*** *=* ***76°***

***10.*** *The points* ***P(−2, −1), Q(h, 7)*** *and* ***R(−3, 6)*** *lie on a circle with diameter* ***PQ.***

***(i)*** *State with a reason the size of angle* ***PRQ***

***(ii)*** *Show that* ***h = 4***

***(iii)*** *Find the coordinates of the centre and radius of the circle*

***Soln:***

***R(−3, 6)***

***P(−2, −1)***

***Q(h, 7)***

***(i)***

***∠PRQ*** *=* ***90°*** *(angle in a semi−circle)*

***(ii)******Hint:*** *This could be done using gradient method*

*Gradient of* ***PR =*** 

*Gradient of* ***QR =*** 

*For perpendicular lines****,*** 

***∴ h = 4***

***(iii)******Hint:*** *Centre is the midpoint of the diameter and radius is half the diameter*

***Centre*** *=*

***Radius*** *=* 

***EER:***

***1.*** *The vertices of an equilateral triangle* ***A, B*** *and* ***C*** *lie on a circle of radius* ***8cm.*** *Find the****:***

***(i)*** *distance of any side of the triangle from the centre of the circle*

***(ii)*** *length of the side of the triangle*

***2.*** *The vertices of an equilateral triangle of side* ***12cm*** *lie on a circle****.*** *Find the****:***

***(i)*** *radius of the circle*

***(ii)*** *distance of any side of the triangle from the centre of the circle*

***(iii)*** *area of the segments cut off by the triangle*

***3.*** *In the circle below****, BC = BD, BE= DE*** *and* ***∠ABC = 140°.*** *Find the size of angle* ***EDB***

***140°***

***A***

***B***

***C***

***D***

***E***

***4.*** *In the circle below* ***O*** *is its centre and* ***∠AOB = 80°.*** *Find the size of the angles marked* ***f*** *and* ***g***

***80°***

***O***

***A***

***B***

***C***

***f***

***g***

***5.*** *In the circle below* ***O*** *is its centre and* ***∠AOC = 150°.*** *Find the size of angle marked* ***x***

***B***

***C***

***A***

***O***

***140°***

***x***

***6.*** *In the circle below* ***O*** *is its centre****, AO*** *is parallel to* ***BC*** *and* ***∠OBC = 60°.***

***60°***

***O***

***A***

***B***

***C***

*Find the size of angle****:***

***(i)******AOB******(ii)******ACB******(iii)******CAB***

***7.*** *In the circle below* ***O*** *is its centre****, ∠AOB = 110°*** *and* ***∠OAC = 20°.*** *Find the size of angle marked* ***f***

***C***

***B***

***A***

***O***

***f***

***20°***

***110°***

***8.*** *In the figure below* ***AB*** *is the diameter of a circle centre* ***O.******∠BAC = 18°*** *and* ***∠BEC = 50°.*** *Find the size of the angles marked* ***x, y*** *and* ***z***

***A***

***B***

***C***

***D***

***E***

***x***

***y***

***z***

***50°***

***18°***

***•***

***O***

***9.*** *In the figure below* ***AB*** *is the diameter of a circle centre* ***O.******∠APQ = 35°*** *and* ***∠ABR = 22°.***

***•***

***O***

***A***

***B***

***P***

***Q***

***R***

***22°***

***35°***

*Find the size of angle****:***

***(i)******ABQ******(ii)******QRA******(iii)******AOR***

***TANGENT PROPERTIES***

|  |  |
| --- | --- |
| ***Tangent diagrams*** | ***Circle theorems*** |
|  | *The angle between a tangent and the radius is* ***90°*** |
|  | *Tangents from an external point are equal in length* |
| ***x***  ***x*** | *The angle between a tangent and a chord is equal to the angle the chord subtends in the opposite segment (alternate segment)* |

***EXAMPLES:***

***1.*** *A tangent from* ***T*** *to a circle****,*** *centre* ***O*** *and radius* ***7cm*** *touches the circle at* ***P.*** *If* ***OT = 25cm,*** *find the****:***

***(i)*** *length of* ***PT***

***(ii)*** *size of angle* ***PTO***

***(iii)*** *area of* ***PTO*** *that lies outside the circle*

***Soln:***

***(i)***

***O***

***P***

***T***

***θ***

***7cm***

***25cm***

***∠OPT*** *=* ***90°*** *(angle between a tangent and the radius)*

***∴*** **

***(ii)*** 

***∴ θ*** *=* ***16⋅26°***

***(iii) POT*** *= 180° − (90° + 16⋅26°) =* ***73⋅74°*** *(angles in a triangle)*

***∴*** *Required area* 

***2.*** *In the circle below* ***O*** *is its centre****. AB = 12cm*** *is a tangent to the circle at* ***A*** *and* ***∠OBA*** *=* ***30°.***

***30°7***

***B***

***A***

***12cm***

***O***

*Find the****:***

***(i)*** *length of* ***OB***

***(ii)*** *radius of the circle*

***Soln:***

***(i) ∠OAB*** *=* ***90°*** *(angle between a tangent and the radius)*

*⇒*

***∴ OB*** *=* ***13⋅8564cm***

***(ii)*** 

***∴ OA*** *=* ***6⋅9282cm***

***3.*** *In the circle below* ***O*** *is its centre****. AB*** *and* ***CB*** *are tangents to the circle and* ***∠ABC*** *=* ***56°.***

***A***

***B***

***C***

***O***

***D***

***56°***

*Find the size of angle* ***ADC***

***Soln:***

***∠AOC*** *+**90° + 90° + 56° = 360° (angles in a quadrilateral)*

***∴ ∠AOC*** *=* ***124°*** *(angles in a triangle)*

*If ∠AOC = 2∠ADC*

***⇒*** *124° = 2∠ADC*

***∴ ∠ADC*** *=* ***62°***

***4.*** *In the circle below* ***O*** *is its centre****. PT*** *and* ***PS*** *are tangents to the circle of radius* ***6cm*** *and* ***∠POT*** *=* ***50°.***

***50°***

***6cm***

***O***

***P***

***S***

***T***

*Find the area of the shaded region*

***Soln:***

***(i) ∠OTP*** *=* ***90°*** *(angle between a tangent and the radius)*

*⇒*

***∴ PT*** *=* ***7⋅1505cm***

***∴*** *Shaded area* 

***5.*** *In the circle below* ***O*** *is its centre****. AP*** *is a tangent to the circle and* ***∠PAB*** *=* ***68°.***

***C***

***68°***

***P***

***A***

***B***

***•***

***O***

***x***

***y***

*Find the size of the angles marked* ***x*** *and* ***y***

***Soln:***

***x*** *=* ***68°*** *(angle in alternate segments)*

***∠BAC*** *=* ***90°*** *(angle in a semi−circle)*

***∴ y*** *= 180° − (90° + 68° ) =* ***22°*** *(angles in a triangle)*

***6.*** *In the circle below* ***AP*** *is a tangent to the circle and* ***∠PAC*** *=* ***120°.***

***D***

***C***

***B***

***A***

***x***

***y***

***120°***

***P***

*Find the size of the angles marked* ***x*** *and* ***y***

***Soln:***

***x*** *=* ***120°*** *(angle in alternate segments)*

***Also:***  *y + 120° = 180° (angles in a cyclic quadrilateral)*

***∴ y*** *=* ***60°***

***EER:***

***1.*** *The tangents at* ***A*** *and* ***B*** *on a circle of radius* ***7cm*** *intersect at* ***T,*** *and* ***C*** *is any point on the major arc* ***AB.*** *If* ***∠ATB*** *=* ***48°,*** *find the****:***

***(i)*** *size of angle* ***ACB***

***(ii)*** *area bounded by the tangents and the minor arc* ***AB***

***2.*** *In the circle below* ***O*** *is its centre****. BC*** *is a tangent to the circle and* ***∠BAO*** *=* ***20°.***

***20°***

***y***

***x***

***C***

***B***

***A***

***O***

*Find the size of the angles marked* ***x*** *and* ***y***

***3.*** *The angles of a triangle are* ***50°, 60°*** *and* ***70°,*** *and a circle touches the sides at* ***A, B*** *and* ***C.*** *Find the angles of triangle* ***ABC***

***4.*** *In the circle below* ***O*** *is its centre****. AB*** *and* ***CB*** *are tangents to the circle of radius* ***7cm*** *and* ***∠ABC*** *=* ***80°.***

***A***

***B***

***C***

***O***

***f***

***g***

***80°***

***7cm***

*Find the****:***

***(i)*** *size of the angles marked* ***f*** *and* ***g***

***(ii)*** *size of the reflex angle* ***AOC***

***(iii)*** *area bounded by the tangents and the minor arc* ***AC***

***5.*** *In the circle below* ***O*** *is its centre****.******DC*** *is a tangent to the circle* ***∠BAO*** *=* ***28°***

*and* ***∠ACD = 70°***

***70°***

***28°***

***O***

***C***

***B***

***D***

***x***

***y***

***A***

*Find the size of the angles marked* ***x*** *and* ***y***

***6.*** *In the circle below* ***ED*** *is a tangent to the circle****,******AB = AD, ∠BDC = 74°***

*and* ***∠DBC = 35°***

***74°***

***35°***

***C***

***B***

***A***

***E***

***D***

*Find the size of angle****:***

***(i)******DAB******(ii)******BDE******(iii)******DBA******(iv)******EDA***

***7.*** *In the circle below* ***AT*** *is a tangent to the circle****,******AB = CB*** *and* ***∠BAT = 74°.*** *Find the size of the angles marked* ***x*** *and* ***y***

***A***

***B***

***C***

***T***

***74°***

***x***

***y***

***CIRCLES AND SIMILAR TRIANGLES***

|  |  |  |
| --- | --- | --- |
| ***Circle diagrams***  ***A***  ***B***  ***C***  ***D***  ***P*** | ***Analysis diagrams*** | ***Conclusion*** |
| ***‘‘*** *intersecting chords theorem* ***”*** | ***C***  ***D***  ***B***  ***P***  ***A***  ***•***  ***•***  ***\*•***  ***\*•*** | *Triangles* ***ACP*** *and* ***PBD*** *are similar* ***⇒***  ***∴***  ***‘‘*** *Each length is measured from**the meeting point****”*** |
| ***D***  ***E***  ***C***  ***B***  ***A***    ***‘‘*** *intersecting secants theorem* ***”***  ***D***  ***C***  ***B***  ***A*** | ***D***  ***E***  ***C***  ***B***  ***A***  ***•***  ***•***  ***\**** | *Triangles* ***ACD*** *and* ***ECB*** *are similar* ***⇒***  ***∴***  ***‘‘*** *Each length is measured from**the meeting point****”*** |
| ***‘‘*** *intersecting tangent− secant theorem* ***”*** | ***D***  ***C***  ***B***  ***A***  ***•***  ***•***  ***\**** | *Triangles* ***ABC*** *and* ***ABD*** *are similar* ***⇒***  ***∴***  ***‘‘*** *Each length is measured from**the meeting point****”*** |

***EXAMPLES:***

***1.*** *In the circle below chords* ***AB*** *and* ***CD*** *intersect at* ***P.*** *If* ***CP = 3cm, DP = 8cm*** *and* ***AB = 10cm,*** *find the length of* ***AP***

***P***

***D***

***C***

***B***

***A***

***8cm***

***3cm***

***Soln:***

*If AP = x, PB = 10 − x*

*Using*  *(intersecting chords theorem)*

***⇒*** *x(10 − x) = 3(8)*

**

**

***∴ x*** *= 6cm or 4cm*

***2.*** *In the circle below chords* ***AB*** *and* ***ED*** *are produced to intersect at* ***C.*** *If* ***CD = 4cm, ED = 5cm, AB = 9cm*** *and* ***BC = xcm,***

***E***

***D***

***C***

***B***

***A***

***5cm***

***4cm***

***9cm***

***xcm***

*find the****:***

***(i)*** *value of* ***x***

***(ii)*** *ratio of the areas of triangle* ***ACE*** *to that of* ***BCD***

***(iii)*** *area of* ***ABDE*** *if the area of triangle* ***ACE*** *is* 

***Soln:***

***(i)*** *Using*  *(Each length from* ***C****)*

***⇒*** *x(x + 9) = 4(9)*

**

**

***x*** *= 3 or −12*

***∴ x*** *= 3*

***(ii)*** *Using similar triangles* ***ACE*** *and* ***BCD***

***B***

***C***

***D***

***•***

***\****

***3cm***

***4cm***

***E***

***C***

***A***

***•***

***\****

***9cm***

***12cm***

***54cm2***



***∴*** *Required ratio =* ***9:1***

***(iii)*** 

***∴*** *Area* ***BCD*** *=* 

***⇒*** *Required area* 

***3.*** *In the circle below secant* ***ABC*** *intersects tangent* ***AP*** *at* ***A.*** *If* ***AP= 6cm, AB = xcm*** *and* ***BC = (x + 1)cm,*** *find the value of* ***x***

***A***

***B***

***C***

***P***

***6cm***

***x***

***x + 1***

***Soln:***

*Using*  *(Each length from* ***A****)*

***⇒*** *x(2x + 1) =* 

**

**

***x*** *= 4 or −4⋅5*

***∴ x*** *= 4*

***4.*** *In the circle below* ***OL= 4⋅5cm, PM = 3cm, NM = 4cm*** *and* ***LN = 7⋅5cm.***

***L***

***M***

***N***

***P***

***O***

*Find the****:***

***(i)*** *lengths of* ***ON***  *and* ***OP***

***(ii)*** *radius of the circle*

***(iii)*** *area of* ***OLMP***

***Soln:***

***(i)*** *Using similar triangles* ***PMN*** *and* ***OLN,***

***\****

***•***

***4cm***

***3cm***

***M***

***N***

***P***

***7⋅5cm***

***L***

***N***

***O***

***•***

***\****

***4⋅5cm***

***⇒***

***∴ ON*** *=* ***6cm***

*Also****:*** 

***∴ PN*** *=* ***5cm***

***⇒ OP = ON − PN*** *= 6 − 5* ***= 1cm***

***(ii) PMN*** *is a right triangle based on its dimensions* ***(∠PMN*** *=* ***90°)***

***⇒ ∠POL*** *=* ***90°,*** *thus* ***LP*** *is a diameter* ***(****angle in a semi−circle****)***

***∴*** **

***⇒*** *Required radius* 

***(iii)*** *Required area* 

***EER:***

***1.*** *In the circle below chords* ***AB*** *and* ***CD*** *intersect at* ***P.*** *If* ***CP = 3cm, DP = 8cm*** *and* ***AP = 6cm,***

***P***

***D***

***C***

***B***

***A***

***8cm***

***3cm***

***6cm***

*find the:*

*(i) length of* ***PB***

***(ii)*** *area of triangle* ***BPC*** *if the area of triangle* ***APD*** *is* 

***(iii)*** *ratio of the areas of triangle* ***APC*** *to that of* ***BPD***

***2.*** *In the circle below* ***TE = 12cm*** *is a tangent to the circle at* ***E.*** *Chords* ***AB*** *and* ***CD*** *intersect at* ***P*** *and* ***AB*** *is produced at* ***T.*** *If* ***AP = 6cm, PB = 4cm, BT = xcm, CP = ycm*** *and* ***DP = 8cm,***

***P***

***B***

***A***

***E***

***x***

***6cm***

***4cm***

***8cm***

***12cm***

***T***

***C***

***D***

***y***

*find the****:***

***(i)*** *values of* ***x*** *and* ***y***

***(ii)*** *ratio of the areas of triangle* ***APD*** *to that of* ***BPC***

***3.*** *In the circle below* ***TE*** *is a tangent to the circle at* ***E.*** *Chords* ***AB*** *and* ***CD*** *are produced to intersect at* ***T.*** *If* ***AB = 5cm, DC = 9cm*** *and* ***CT = 3cm,*** *find the lengths of* ***BT*** *and* ***ET***

***3cm***

***9cm***

***5cm***

***E***

***D***

***C***

***B***

***A***

***T***